SPACETIME AT THE PLANCK SCALE: THE QUANTUM COMPUTER VIEW

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Abstract. We assume that spacetime at the Planck scale is discrete, quantised in Planck units, and "qubitised" (each pixel of Planckian area encodes one qubit). Then, we formulate the Quantum Computer View of quantum spacetime. Within this model, one finds that quantum spacetime might be in a entangled state, and might quantum-evaluate Boolean functions which are the laws of Physics in their most fundamental form.

1. INTRODUCTION

What is "spacetime" at the Planck scale? Once we understand that, we will be able to formulate the theory of Quantum Gravity, the theory which should reconcile General Relativity and Quantum Mechanics. In fact, it is widely believed that at the Planck scale, the quantum aspects of gravity become relevant. Moreover, it is generally assumed that at the Planck scale, spacetime is not any longer a smooth manifold, but has a discrete structure.

There are two main approaches to quantum gravity that assume quantum spacetime to be discrete: Loop Quantum Gravity [1] (and spin foams [2]), and String (and M) Theory [3]. Other interesting approaches are non-commutative geometry [4], Causal Set Theory [5] and kinds of discrete models of spacetime at the Planck scale, like lattice versions of loop quantum gravity [6], and Cellular Networks [7].

In our particular approach to quantum gravity, we assume discreteness of spacetime at the Planck scale, and we also include the issue of information, (more precisely quantum information [8]). In fact, as it was suggested by Wheeler (the "It from bit" proposal) [9], information theory must play a relevant role in understanding the foundations of Quantum Mechanics. Wheeler's view is shared, in particular, by Zeilinger (who associates bits with elementary systems, i.e. two-level systems, and claims that the world appears quantised because information is quantised) [10] and by Deutsch (who says that "quantum computing *is* quantum mechanics"). As it was first realized by Feynman, a quantum computer can be exponentially more powerful than a classical one in simulating a quantum system. This line of thought is what we call here the "Quantum Computer View" (QCV).

We believe that the QCV is universal, and thus can be extended to the "description" of quantum spacetime itself. Approaches similar to ours, still encompassing the QCV, are those of Lloyd [11], Zimmermann [12], Hitchcock [13] and Jaroszkiewicz [14]. Our approach, (as well as the one of Zimmermann) is closely related to Loop Quantum gravity and spin networks [15] [16]. Spin networks are relevant for quantum geometry. They were invented by Penrose [15] in order to approach a drastic change in the concept of spacetime, going from that of a smooth manifold to that of a discrete, purely combinatorial structure. Then, spin networks were re-discovered by Rovelli and Smolin [16] in the context of loop quantum gravity [1]. Basically, spin networks are graphs embedded in 3-space, with edges labelled by spins and vertices labelled by intertwining operators. In loop quantum gravity, spin networks are eigenstates of the area and volume-operators [17].

We interpret spin networks as qubits when their edges are labelled by the spin-1/2 representation of SU(2). In this context, we use the quantum version [18] of the Holographic Principle [19]. In our model, quantum spacetime is discrete, quantised in Planck units, and each pixel of Planckian area, encodes a qubit (qubitisation [20] of quantum spacetime). This is a quantum memory register. To process the quantum information stored in the memory, it is necessary to dispose of a network of quantum logic gates (which are unitary operators). The network must be part of quantum spacetime itself, as it describes its dynamical evolution. The quantum memory plus the quantum network form a quantum computer (quantum computer view of quantum spacetime).
In the QCV, some new features of quantum spacetime emerge:

- The dynamical evolution of quantum spacetime is a reversible process, as it is described by a network of unitary operators. This leads to some weird consequences…

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- During a quantum computational process, quantum spacetime can be in a entangled state, which leads to non-locality of spacetime itself at the Planck scale (all pixels are in a non separable state, and each pixel loses its own identity).
- As entanglement is a particular case of superposition, quantum spacetime is in a superposed state, which is reminiscent of the Many-Worlds interpretation of Quantum Mechanics [21].
- Due to superposition and entanglement, quantum spacetime can compute a Boolean function for all inputs simultaneously (massive quantum parallelism). We argue that the functions which are quantum-evaluated by quantum spacetime are the laws of Physics in their most fundamental, discrete and abstract form. Moreover, as the laws are the outputs of quantum measurements, their origin is probabilistic.
- By scratch space management, we find that at the Planck scale it is possible to compute composed functions of maximal depth.
- The quantum information stored and processed by quantum spacetime prevents direct tests of the Planck scale.

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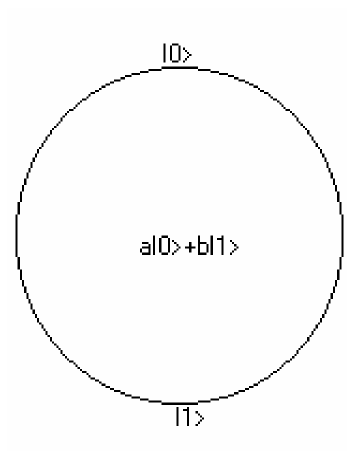
Fig. 1: The Bloch sphere



2. QUBITISATION OF QUANTUM SPACETIME

The very concept of event should be revised in the context of quantum spacetime. In fact, the definition of event as a point in a four-dimensional smooth manifold becomes meaningless once spacetime is assumed to be discrete, and quantized in Planck units. If the minimal length is assumed to be the Planck length: $l_P \approx 10^{-33} cm$ and the minimal time interval is assumed to be the Planck time: $t_P \approx 10^{-43}\sec$, it follows that an event in quantum spacetime is an extended object without structure (a block). In the QCV, the quantum event encodes quantum information.

The (classical) holographic principle [19] states that it must be possible to describe all phenomena within the bulk of a region of space of vo lume V by a set of degrees of freedom which reside on the boundary, and that this number should not be larger than one binary degree of freedom per Planck area. All this can be interpreted as follows: each unit of Planck area (a pixel) is associated with a classical bit of information.

At the Planck scale, however, where quantum gravity takes place, we argue that the encoded information should be quantum, and the holographic principle should be replaced by its quantum version [18]. In the quantum version of the holographic principle, a pixel encodes one quantum bit (qubit) of information. (A qubit is a linear superposition of the logical states 0 and 1, namely: $|Q\rangle = a|0\rangle + b|1\rangle$, where a and b are complex numbers called probability amplitudes, such that $|a|^2 + |b|^2 = 1$).

The necessity of the quantum version of the holographic principle follows directly from loop quantum gravity. In loop quantum gravity, non-perturbativa techniques have led to a quantum theory of geometry in which operators corresponding to lengths, area and volume have discrete spectra. Of particular interest are the spin network states associated with graphs embedded in 3-space with edges labelled by spins

$$j = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$$

and vertices labelled by intertwining operators.

If a single edge punctures a 2-surface transversely, it contributes an area proportional to [17]:

$$l_P^2 \sqrt{j(j+1)}$$

Let us consider the edges of spin networks in the spin -1/2 representation of SU(2): they are 2-level systems, and can be thought as qubits. In mathematical terms, the group manifold of $SU(2)$ can be parameterized by a 3-sphere with unit radius. In fact, the most general form of 2×2 unitary matrices of unit determinant is:

$$U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \qquad |a|^2 + |b|^2 = 1$$

where a and b are complex numbers.

For example, the action of the unitary SU(2) matrix $U_{s_2} = \frac{1}{\sqrt{2}}(1 + i s_2)$, where s_2 is the Pauli matrix: $s_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ on the edge states $\left|-\frac{1}{2}\right\rangle$ and $\left|+\frac{1}{2}\right\rangle$ respectively, gives the equally superposed states

$$\frac{1}{\sqrt{2}}\left(\left|\frac{1}{2}\right\rangle \pm \left|-\frac{1}{2}\right\rangle\right).$$

When a surface is punctured by such a superposed state, a pixel of area is created, which encodes a qubit (qubitisation [20] of spacetime at the Planck scale).

The elementary (Plankian) pixel can then be viewed as the surface of a unit (in Planck units) sphere in three dimensions. The pixel is punctured (simultaneously) in the poles by an edge in the superposed state of spin down and spin up.

Equivalently, a qubit corresponds to the surface of the 3-dimensional unit sphere, where the logic states 0 and 1 correspond to the poles. This is the so-called Bloch sphere (see fig. 1).

There is clearly an analogy between the spin networks approach to quantum gravity and our Quantum Computer View of quantum spacetime.

3. IS QUANTUM SPACETIME A QUANTUM COMPUTER?

Having assumed that spacetime at the Planck scale encodes quantum information, the latter must be processed to give rise, as an output, to the universe as we know it. If so, quantum spacetime is not just a quantum memory register of n qubits: it is the whole thing, a quantum memory register plus a network of quantum logic gates. In other words, spacetime at the Planck scale must be in such a quantum state to be able to evaluate those discrete functions which are the laws of Physics in their discrete and most fundamental form. We may interpret that quantum state as the state of a quantum computer which is computing Boolean functions. But doing so, we should assume that at the Planck scale spacetime is in a superposed/entangled state. In fact, any efficient quantum algorithm relies on superposition and entanglement of qubits. In quantum computation, superposition and entanglement are very important, because they allow quantum parallelism: the possibility to compute exponentially many values of a function in polynomial time. (Entanglement is a special case of superposition, which has no classical analogue: an entangled state is non-separable).

4. IS QUANTUM SPACETIME IN A SUPERPOSED/ENTANGLED STATE?

If the qubits encoded by pixels were superposed, the surface embedding a region of space would "exist" in many different states simultaneously. This would be a quite weird wave-like aspect of quantum spacetime itself. Superposition is one characteristic feature of quantum mechanics, but we should be aware of the fact that once applied to quantum spacetime, it spoils the latter of the usual attributes. We think that the idea of a superposed state of qubits associated to pixels, fits quite well in the Many-Worlds interpretation of Quantum Mechanics, obviously restricted to the micro-domain of spacetime itself, more precisely at the fundamental level.

Spacetime at the Planck scale, like the state of a quantum computer, can only decohere at the end of every computational process, which terminates with a measurement. In the case of quantum spacetime, we should better say that decoherence is due to "self-measurement" (projection operators must be included in the quantum spacetime structure).

Do the pixels of Planckian area encode qubits which are entangled to each other or not? In the affirmative, spacetime itself would be spoiled of locality, at the Planck scale. In other words, two quantum events might be described by a single quantum state, each event losing its own identity. This would be a quite weird feature of quantum spacetime, but it cannot be discarded *a priori*, because entanglement is a very peculiar feature of the quantum world.

Let us consider a finite number N of pixels p_i (i=1,2…N) each one encoding one qubit $|Q\rangle_i$ (notice that the number of pixels of area of a certain surface S is equal to the number of punctures made by spin network' edges in the 1/2 -representation of SU(2) onto S). The N qubits span a Hilbert space of dimension 2^N. The standard basis for one-qubit is: $|0\rangle, |1\rangle$. The dual basis for one-qubit is:

$$\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

The most general one-qubit is: $|Q\rangle = a|0\rangle + b|1\rangle$ where a and b are complex numbers such that:

$$|a|^2 + |b|^2 = 1$$

A 2-qubits state can be either unentangled (product state of two 1-qubit states) or entangled (a non-separable state). The unentangled basis for two qubits is:

$$|00\rangle, |01\rangle, |10\rangle, |11\rangle$$

An example of unentangled 2-qubit state is the product of one dual basis vector and the qubit $|0\rangle$:

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

The entangled basis for 2-qubits (Bell states, maximally entangled) is:

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|10\rangle \pm |01\rangle), \qquad |\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|11\rangle \pm |00\rangle)$$

5. SUPERPOSITION AND ENTANGLEMENT OF PIXELS DEPEND ON THE QUANTUM NETWORK CHOSEN BY NATURE

Let us suppose that all the N qubits encoded by N pixels are initially in state $|000.....0\rangle$. They form a quantum register of size N, but that is just a storage of quantum information. To be able to perform quantum computation, the qubits of the memory must be manipulated by some unitary transformations performed by quantum logic gates (the number of the gates is called the size of the network). Now, to make a superposition of 2 qubits, it is necessary to dispose of the Hadamard gate:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

and to entangle 2 qubits, it is necessary the controlled - NOT (or XOR) gate:

$$XOR = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

In the case of n qubits, we need the Walsh-Hadamard transformation:

$$H_n = H^{\otimes n}$$

Let us see how it works in the case of two qubits. Let us write the standard basis in vector notation:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The action of the Hadamard gate on the ket $|0\rangle$ is:

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

and on the ket $|1\rangle$ is:

$$H|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Consider a quantum register of size two in state $|00\rangle$. The action of the Hadamard gate H on the first qubit gives the superposed state:

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

If we take the superposed state as the control qubit (c), and the second qubit of the memory as the target qubit (t), the action of the XOR gate is:

$$XOR: \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_{(c)}|0\rangle_{(t)} \rightarrow \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

which is an entangled state of two qubits.

A quantum memory register of size n is a collection of n qubits. Information is stored in the quantum register in binary form. The state of n qubits is the unit vector in the 2^n-dimensional complex Hilbert space: $C^2 \otimes C^2 \otimes ... \otimes C^2$ n times. As a natural basis, we take the computational basis, consisting of 2^n vectors, which correspond to 2^n classical strings of length n:

$$|0\rangle \otimes |0\rangle \otimes ... \otimes |0\rangle \equiv |00...0\rangle$$
$$|0\rangle \otimes |0\rangle \otimes ... \otimes |1\rangle \equiv |00...1\rangle$$
$$\vdots$$
$$|1\rangle \otimes |1\rangle \otimes ... \otimes |1\rangle \equiv |11...1\rangle$$

In general, we will denote one basis vector of the state of n qubits as:

$$|x_1\rangle \otimes |x_2\rangle \otimes ... \otimes |x_n\rangle \equiv |x_1 x_2 ... x_n\rangle \equiv |x\rangle$$

where $x_1, x_2, ..., x_x$ is the binary representation of the integer x, a number between 0 and 2^{n-1}.

The general state is a complex unit vector in the Hilbert space, which is a linear superposition of the basis states:

$$\sum_{x=0}^{2^n-1} c_i |x\rangle$$

where c_i are the complex amplitudes of the basis states $|i\rangle$, with the condition: $\sum_i |c_i|^2 = 1$

To perform computation with n qubits, we have to use quantum logic gates. A quantum logic gate on n qubits is a $2^n \times 2^n$ unitary matrix U. Initially, all the qubits of a quantum register are set to $|0\rangle$. By the action of the Walsh-Hadamard transform, the n input qubits are set into an equal superposition:

$$\frac{1}{\sqrt{2}^n}\sum_{x=0}^{2^n-1}|x\rangle$$

At this point the very computation can start.

6. QUANTUM FUNCTION EVALUATION AT THE PLANCK SCALE

The quantum computation of Boolean functions f is implemented by unitary operators U_f. In the case of bijective functions $f:\{0,1\}^n \to \{0,1\}^n$, which are reversible, it always exists a unitary operator U_f such that:

$$U_f : |x\rangle \to |f(x)\rangle.$$

where $|x\rangle$ stands (for brevity) for the input register, namely $\frac{1}{\sqrt{2}^n}\sum_{x=0}^{2^n-1}|x\rangle$.

The quantum computation of non bijective functions f: $\{0,1\}^n \to \{0,1\}^m$, (which are non reversible) requires (at least) two registers, in order to guarantee the unitarity of U_f (reversibility of the computation): a register of size n to keep a copy of the arguments of f, and a second register of size m, to store the values of f: $U_f|x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle$, where $\oplus$ stands for addition mod 2^m.

Notice that in general, (for non trivial functions) the states $|x\rangle|y \oplus f(x)\rangle$ are entangled.

Moreover, the quantum computation of f on a superposition of different inputs, produces f(x) for all x in a single run (quantum parallelism):

$$\sum_x |x\rangle|0\rangle \to \sum_x |x\rangle|f(x)\rangle$$

But we cannot get all values of f(x) from the entangled state $\sum_x |x\rangle|f(x)\rangle$ as any measurement on the first register will yield one particular value x', and the second register will then be found with the value f(x'). It is possible, however, to compute some global properties of f(x) in a single run. As we already said, both superposition and entanglement are necessary for quantum computation. But it is not obvious that quantum information stored in quantum spacetime is exploited to perform quantum computation. It depends on which kind of quantum network (if any) Nature has chosen.

The question is: what should be computed by quantum spacetime? The answer is: the global properties of Boolean functions, as in a quantum computer. In our case, we argue that the output of quantum computation would be the global structure of the Laws of Physics. However, as we have seen, the output (the result of a measurement) appears randomly, thus the nature of the global properties of such laws is probabilistic.

In fact, while the whole quantum computational process is deterministic, in the sense that time evolution is guaranteed by a unitary operator, the output is random, as measurement is a non-unitary operation. Some extra registers (called scratch space) are also needed to store intermediate results. In longer calculations (for example in computing composite functions) this leads to a large amount of "garbage" (or "junk") qubits, which are not relevant to the final result. In order not to waste space, these "junk" qubits must be re-set to $|0\rangle$ and the scratch space can then be "recycled" for further computations. Scratch space management was proposed by Bennett [22].

Let us suppose we have to calculate a composite function of depth d. Without scratch space management, the computation would need d operations, and would consume d-1 junk registers. With scratch space management, the computation will need 2d-1 operations, and d-1 scratch registers. For example, the computation of a composite function of depth d = 2, f(x) = h(g((x))), would need 3 operations, and one scratch register, which can be reused in further computation:

$$\left|x,0,0\right\rangle \xrightarrow{U_{g12}} \left|x,g(x),0\right\rangle \xrightarrow{U_{h23}} \left|x,g(x),h(g(x))\right\rangle \xrightarrow{U_{g12}^{+}} \left|x,0,f(x)\right\rangle$$

Where U_g, U_h are the unitary operators implementing the quantum computations of functions g and h respectively, and the suffix numbers refer to the registers operated on. The last step of the computation, is just the inversion of the first step and uncomputes the intermediate result. The second register can then be reused for further computations.

As we have seen, the number of required scratch registers, increases linearly with the depth of the composite function which has to be quantum computed. This fact will be very useful to our purpose.

We can imagine the boundary surface S enclosing a volume V of space, as a collection of N pixels of Planckian area, each encoding a qubit. Thus S is a quantum memory register of N qubits. If all N qubits are initially set to $\left|0\right\rangle$, as always before any computation, the original register can be thought as the product of several registers: $\left|0\right\rangle_x \left|0\right\rangle_y \left|0\right\rangle_z ....\left|0\right\rangle_w$ where registers x, y, z…w have respectively size n, m, k,…, r such that n+m+k…+ r = N.

The initial quantum state $\left|\Psi\right\rangle \in C^{2^N}$ of S is then:

$$\left|\Psi\right\rangle = \left|0\right\rangle_x \left|0\right\rangle_y \left|0\right\rangle_z ....\left|0\right\rangle_w$$

Suppose that register $\left|0\right\rangle_x$ has the smallest size, for example n=2. This size is very close to the Planck scale, as for

n = 2, it is $l^2 = 4l_P^2$.

The register $\left|0\right\rangle_x$ can be set to an equal superposition of basis states by the action of the Walsh-Hadamard transform $H^{\otimes 2}$ which acts locally on it:

$$H^{\otimes 2}\left|0\right\rangle_x = \frac{1}{2}\sum_{x=0}^{3}\left|x\right\rangle$$

Now the quantum state of S is:

$$\left|\Psi\right\rangle' = \left|x\right\rangle\left|0\right\rangle_y \left|0\right\rangle_z .....\left|0\right\rangle_w, \text{ where } \left|x\right\rangle \text{ stands for } \frac{1}{2}\sum_{x=0}^{3}\left|x\right\rangle.$$

In our case, the quantum computation of a function f: $\{0,1\}^n \to \{0,1\}^{N-n}$ can be implemented by a unitary operator such that: $U_f : \sum_x \left|x\right\rangle\left|0\right\rangle_y \to \sum_x \left|x, f(x)\right\rangle$ only if the second register y has the right size to accommodate f, i.e, m=N-n, and there are no other registers available. However, if the computation of f produces n' junks bits which fill a scratch register of size n', a second register of size n', has to be provided. The best way to solve this problem, is to take a smaller first register x to enable scratch space management. Moreover, if f is a composite function f(h((g(l(….(x))))) of depth d, the original register of size N must be partitioned in such a way that there are d-1 scratch registers available. So, in order to compute highly composite functions, the first register (storing the argument) must have the smallest possible size, to leave room for the needed number of scratch registers. In particular, if n=1 (the Planck scale), the available scratch space has size N-1, and the highest level of composition for f is d=N when d-1 scratch registers, of one qubit each, sum up to the original register of size N. Thus, the quantum computation of highly composite functions must be performed close to the Planck scale, and the output (some global property of f) is obtained at macroscopic scales.

According to inflationary cosmological theories, the cosmological horizon has at present a radius $R \approx 10^{60} l_P$, thus its surface area is $A \approx 10^{120} l_P^2$, that is an area of 10^{120} pixels, each one encoding one qubit. In the QCV, the cosmological horizon's surface can be interpreted as a quantum memory register of $N = 10^{120}$ qubits. Thus, spacetime at the Planck scale can compute a composite function of maximal depth $d = 10^{120}$.

Cosmological models based on the QCV have been considered by the author [23] and by Lloyd [24].

As we have already said, we believe that the recursive functions computed by quantum spacetime at the Planck scale, are the laws of Physics in their discrete, abstract, and fundamental form. We, human beings, who are "derived from the laws", look like fixed points (in the sense of Gödel diagonalization lemma), as we are part of the program and still we are aware that it is running [25] (although we cannot grasp the whole of it).

7. TWO DIFFERENT CONCEPTUAL LEVELS OF PLANCK SCALE PHYSICS

The Heisenberg time-energy uncertainty relation, $\Delta E \Delta t \geq \hbar$, allows virtual particles of mass $m = \Delta E / c^2$ to come into existence for an interval of time $\Delta t \geq \hbar / \Delta E$. The time-energy uncertainty relation is saturated at the Planck scale: $E_P t_P = \hbar$. This means that an object having a Planck mass $m_P = \left(\frac{\hbar c}{G} \right)^{1/2} \cong 10^{-5} g$ and size equal to the Planck length $l_P = c t_P = \left(\frac{G\hbar}{c^3} \right)^{1/2} \cong 10^{-33} cm$, cannot be anything else than a virtual object.

The particle-like character of such a virtual object is enlightened by the fact that the Compton length $l_C = \frac{\hbar}{mc}$, when calculated for the Planck mass, coincides with the Planck length: $l_C(m_P) = l_P$. Moreover, the Schwarzchild radius $l_S = \frac{2Gm}{c^2}$, when calculated for the Planck mass, is twice the Planck length: $l_S(m_P) = 2l_P$. The factor 2 (although very often discarded in the literature [26]) is very important, as the Schwarzchild radius of this object is bigger than its size, thus the object is a black hole.
Then, at the Planck scale, we have virtual particle-like black holes/wormholes, as it was suggested by Wheeler (the quantum foam) [27] and Hawking [28]. Notice, however, that the particle-like structure of such an object is defined at the very Planck length by the Compton length, while its black hole structure is defined at twice the Planck length by the Schwarzchild radius. The surface area of the event horizon of such a virtual Schwarzchild black hole, is about four Planckian pixels, and encodes four qubits.

In our model, lengths are quantised in Planck units: $l_n = n l_P$ and the quantised area is: $A = a\, n^2 l_P^2$. The value of the constant a is fixed to $a = 4\ln 2$ by the Bekenstein formula: $S = \frac{A}{4 l_P^2}$, where $S = N \ln 2$ is the information entropy, and $N = n^2$ is the number of qubits. Then our discrete area spectrum is: $A_n = 4\ln 2 n^2 l_P^2$.
In our case, the area levels are not equi spaced , (differently from Bekenstein's model [29], with area spectrum: $A = 4\ln 2\, n l_P^2$).

For n=1, we get the area of the event horizon of a virtual Schwarzchild black hole $A_1 = 4\ln 2 l_P^2$, which coincides with the minimal eigenvalue of the area spectrum calculated in loop quantum gravity when considered in the context of black hole entropy [30] and in black holes quantisation models [29]. However, in our scheme, it would not be possible to consider particle-like quantum black holes with masses larger than the Planck mass, as in that case the Compton length would be a fraction of the Planck length, which is impossible (for example, a black hole of mass $2m_P$, would have a Compton length $l_C = \frac{1}{2} l_P$, which does not make sense).

Then, black holes with masses larger than the Planck mass (already at twice the Planck mass) would lose any quantum particle-like feature, although they could be considered as the intermediate remnants of evaporating classical (large) black holes. In this sense we disagree with the interpretation of quantum black holes in the existing literature [29], although we agree with the formalism of the quantization of the area of the surface horizon. These topics will be extended in future work [31]. Thus, the Planck mass really sets the scale for quantum gravity, where both particle-like and black hole-like structures can simultaneously exist.

For masses soon above the Planck mass, the Compton length would be at scales below the Planck scale, which would give unphysical results in terms of particle physics, and we are left with the so called quantum black holes, which do not have, however, the features of elementary particles. They might be called quantum only for the fact that their horizon surface area is quantised, just because we are still in the domain of discrete spacetime. At larger scales, (about $n = 10^{20}$), where spacetime has already emerged as a continuum, one can start to speak about quantum field theory for small masses such that $l_C >> l_S$.

To formalize the considerations expressed above, it might be useful to look at the behaviours of the Compton length and of the Schwarzchild radius in function of integer multiples and fractions of the Planck mass respectively:

$$l_C(n\,m_P) = \frac{1}{n} l_P; \qquad l_C\left(\frac{m_P}{n}\right) = n l_P;$$

$$l_S(n\,m_P) = 2n l_P; \qquad l_S\left(\frac{m_P}{n}\right) = \frac{2}{n} l_P$$

Thus, the Compton length does not make sense for integer multiples of the Planck mass, while the Schwarzchild radius does not make sense for fractions of the Planck mass. We believe that, although the area spectra discussed in this paper and in [29] might be useful to enlighten the quantum behaviour of small black holes, they cannot be extended to large black holes, as their associate mass spectra are degenerate: $M \propto n$ in our model, and $M \propto \sqrt{n}$ in [29]. Instead, the loop quantum gravity area spectrum is more adequate for larger black holes, as the associated mass spectrum is consistent with Hawking's radiation at the semiclassical level.

In the QCV, the double nature (particle-like and black hole-like) of the virtual quantum object at the Planck scale means the following. One Planckian pixel represents a qubit, but on its own, it is just the unit of quantum information, and cannot perform any computation (this is the elementary particle aspect of the Planck scale, it is a two-level system, which according to Zeilinger [10] is an elementary system). However, a virtual quantum black hole, at the Planck scale has a horizon surface area encoding four qubits, and can perform quantum computation. Finally, we argue that the intrinsic non local aspect of quantum spacetime at the Planck scale discussed in this paper, might be due to virtual wormholes connecting Planckian pixels, as wormholes violate locality. In this view, virtual wormholes should be considered as tiny XOR gates. In other words, in the QCV, the scale which allows a quantum computing spacetime is the scale of quantum gravity, the Planck scale, which is the seat of quantum foam.

8. UNITARY EVOLUTION AND ITS CONSEQUENCES

The quantum evolution of a quantum computer is described by unitary operators, and this guarantees a reversible computation. It follows that, in the quantum computer view, the dynamical evolution of quantum spacetime itself is a reversible process. This sounds like a paradox, as far as we think of quantum spacetime as a pre-spacetime with almost all the same characteristics of classical spacetime, which is the seat of irreversibility. But the very idea of motion is lost at the Planck scale, so irreversibility does not make sense at that level. Irreversibility might be just an emergent feature at larger scales.

One should be able, however, to figure out what it means reversibility of quantum spacetime itself. The simplest answer leads us back to Wheeler's "spacetime foam" [27], made up of virtual black holes (and wormholes). Like all virtual processes, also this one takes place by virtue of the time-energy uncertainty relation, (which at the Planck scale is saturated). A quantum black hole of Planck mass, comes into existence out of the vacuum, and then evaporates in Planck time, releasing a quantum of Planckian energy back to the vacuum. As this "virtual" process is due to quantum fluctuations of the vacuum, which are non-dissipative [32], it can be considered a reversible process, unless a measurement takes place. But virtual particles cannot be probed. However, the complete evaporation of a black hole is non-unitary as it transforms a pure state in a mixed state [33], and could be interpreted as the act of self-measurement at the end of a quantum computation performed by quantum spacetime. Self-measurement (or self-decoherence) can be realized only in such virtual processes which end by transforming a pure state into a mixed state, with information loss.

In this picture, superposed/entangled pixels would mean an intricate virtual 2-dimensional structure, fluctuating in between existence and non existence at each Planck time step. In this context, the word *existence* is the classical attribute of the output of a quantum computation (a string of bits). The "paradoxial" (from a classical point of view) consequences of the unitary evolution of spacetime at the Planck scale, (reversibility jointly with self-decoherence), just express the simple fact that there is no time flow at the Planck scale (although a vague concept of block-time is already present) and that dynamical evolution in time is meaningless. This might be the source of the problems arising when one tries to formulate the dynamics of loop quantum gravity. What we would like to still call (erroneously) "evolution in time" is in fact a program executed by a network of quantum logic gates, in discrete Planck time steps.

9. OBSTRUCTION TO THE PLANCK SCALE

In string theory, computations of very high scattering amplitudes [34] suggest that geometry below the Planck scale cannot be probed, or, in other words, it does not exist a geometrical sub-Planckian structure of spacetime.
From the QCV of spacetime at the Planck scale discussed in this paper, we arrive at the same conclusion. In fact, looking for the sub-structure of Planck-scale spacetime, would mean searching for the sub-structure of the quantum of information (the qubit) encoded in a pixel of Planckian area. But, if we measure a qubit $|Q\rangle = a|0\rangle + b|1\rangle$, we get $|0\rangle$ with probability $|a|^2$, leaving the state $|Q\rangle' = |0\rangle$ or $|1\rangle$ with probability $|b|^2$ leaving the state $|Q\rangle' = |1\rangle$. Thus, from a single measurement, one obtains only one bit of information. Only from measurements of an infinite number of identically prepared qubits, one would be able to know a and b, i.e. the state $|Q\rangle$.

In this sense, a qubit encodes a lot of "hidden" information, as long as it is not measured, and a great deal of this hidden information is lost once one performs a measurement. What does it mean in "spacetime terms"?
Let us recall that, as discussed in previous section, spacetime at the Planck scale may be viewed as a sea of virtual Planckian black holes. Probing the Planck scale would then mean losing data inside the event horizons of Planckian black holes. Or, even worse, the data might be relative to another world if , in measuring an entangled state, one is faced with a virtual wormhole.

10. CONCLUSIONS

The QCV of spacetime at the Planck scale relies on linear concepts like superposition and entanglement. Thus, this view cannot be extended to the macroscopic domain, where spacetime is described by the non linear equations of General Relativity. To understand how, from the linearity of the Planck scale level we obtain the non linearity of the classical macroscopic level, it might be useful to consider self-organizing models and related technicalities. This is what we call emergence of classicality and complexity (our classical world emerges as one which is complex).
As we have seen, in the QCV, quantum spacetime looks like having a reversible dynamical evolution. But what does it mean that space-(time) evolves in time, and moreover in a reversible manner? As we have seen, this paradox, can be solved by assuming Wheeler's picture of "spacetime foam" which however excludes time flow at the Planck scale.

Thus, both non linearity and irreversibility, which have no home in the QCV, should be emergent features of spacetime. In the QCV, also locality is lost: "spacetime" itself is non local at the Planck scale, due to the entanglement of pixels/qubits. This is very much on line with Penrose's argument, stating that the theory emergent from spin networks should have a fundamentally non-local character [35]. As far as causality is concerned, it is a more subtle point, however we believe that causality is missing at the Planck scale, as time flow is absent.
Despite all these weird aspects, if spacetime at the Planck scale really behaves like a quantum computer, it is able to evaluate the most complex laws of Physics.

We wish to conclude by saying that in the QCV, spin networks, qubits, and virtual black holes are different aspects of the same building blocks of quantum spacetime at the Planck scale. In philosophical terms, spin networks (at least in their original form introduced by Penrose [15] as purely combinatorial objects, without the introduction of causal sets as in [36]), are fundamental in the sense that they are attributes of substance [37]. Spin networks are the boundary (at the Planck scale) between the physical world and its foundations. Thus, in the QCV, also qubit states associated with spin networks are a different way of saying the same thing: quantum information on its own is an attribute of substance. However, when quantum computation is taken into account, and a (reversible) dynamical evolution arises, we are already a little bit upward the boundary between substance and the physical world. Virtual black holes, (and wormholes) constitute mini quantum computers, which "prepare" the physical world.

By resuming, in the QCV, spin networks, qubits and quantum foam are three different aspects of fundamental Physics (at the Planck scale). As none of them is directly testable a priori, it follows that this worldview suggests that fundamental Physics is not empirical in the usual sense. But still, it is not metaphysics, which is substance, without attributes.

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